Comparison of solvers performance when solving the 3D Helmholtz elastic wave equations over the Hybridizable Discontinuous Galerkin method

Marie Bonnasse-Gahot, Henri Calandra, Julien Diaz, Stephane Lanteri

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Motivations

Imaging methods

- Full Wave Inversion (FWI) : **inversion process** requiring to solve many forward problems

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Resolution of the forward problem of the inversion process

- Elastic wave propagation in the frequency domain : Helmholtz equation

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First order formulation of Helmholtz wave equations

$\mathbf{x} = (x, y, z) \in \Omega \subset \mathbb{R}^3$,

$$\begin{cases} i\omega\rho(\mathbf{x})\mathbf{v}(\mathbf{x}) = \nabla\cdot \underline{\underline{\sigma}}(\mathbf{x}) + f_s(\mathbf{x}) \\ i\omega\underline{\underline{\sigma}}(\mathbf{x}) = \underline{\underline{C}}(\mathbf{x})\ \underline{\underline{\varepsilon}}(\mathbf{v}(\mathbf{x})) \end{cases}$$

- $\mathbf{v}$: velocity vector
- $\underline{\underline{\sigma}}$: stress tensor
- $\underline{\underline{\varepsilon}}$: strain tensor

Approximation methods

Discontinuous Galerkin Methods

- ✓ unstructured tetrahedral meshes
- ✓ combination between FEM and finite volume method (FVM)
- ✓ *hp*-adaptivity
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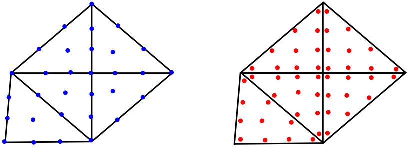

Approximation methods

Hybridizable Discontinuous Galerkin Methods

✓ same advantages as DG methods : unstructured tetrahedral meshes, *hp*-adaptivity, easily parallelizable method, discontinuous basis functions

✓ introduction of a new variable defined only on the interfaces

✓ lower number of coupled DOF than classical DG methods

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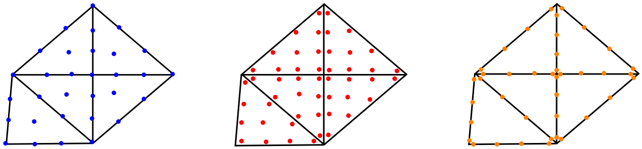

Principles of the HDG method

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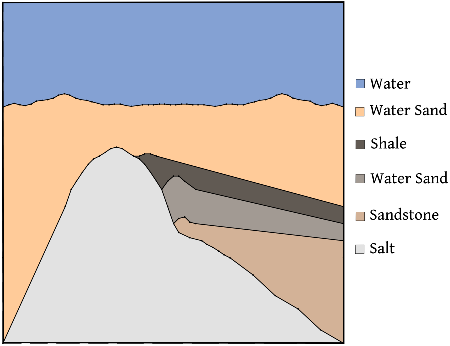
5. Computation of the solutions of the initial problem, element by element

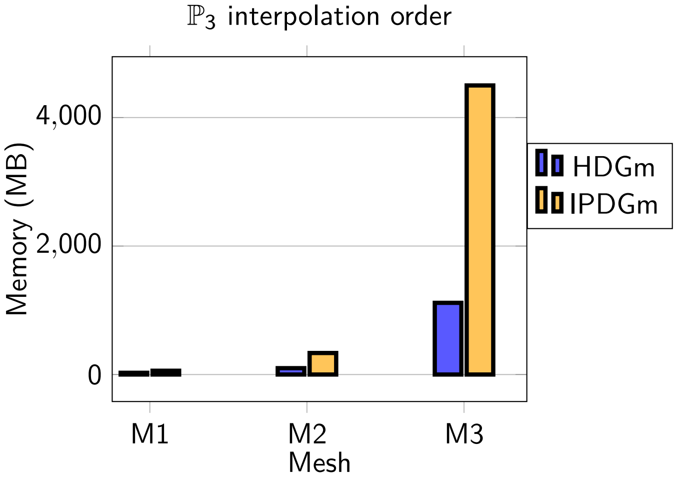
Contents

Anisotropic test case



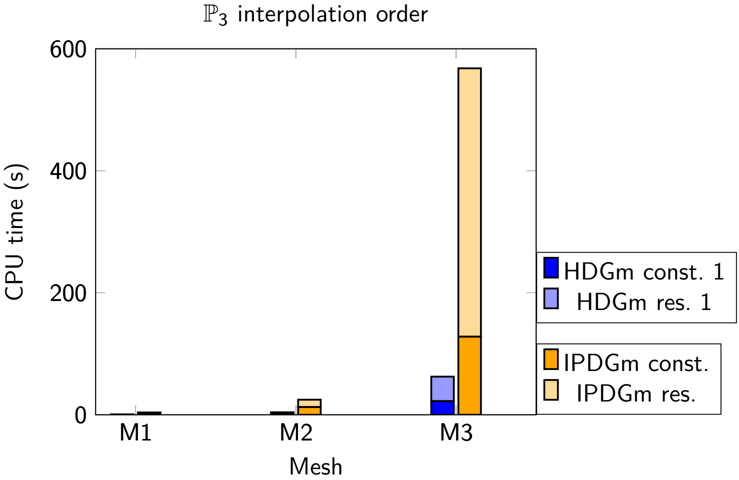
- Three meshes :
  - 600 elements
  - 3000 elements
  - 28000 elements

Anisotropic case : Memory consumption



$\textbf{IPDG}_{\textbf{memory}} \simeq \textcolor{red}{\textbf{4}} \times \textbf{HDG}_{\textbf{memory}}$

Anisotropic case : CPU time (s)



$$\mathbf{IPDG}_{\mathbf{CPUtime}} \simeq \mathbf{9} \times \mathbf{HDG}_{\mathbf{CPUtime}}$$

Contents

Resolution of the linear system $\mathbb{M}\lambda = \mathbb{S}$

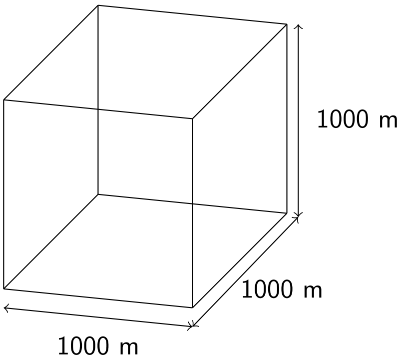
- **direct solver : MUMPS** (MUltifrontal Massively Parallel sparse direct Solver) :
  - Direct factorization $A = LU$ or $A = LDL^T$
  - Multiple RHS
- **hybrid solver : MaPhys** (Massively Parallel Hybrid Solver) :
  - combination of direct and iterative methods
  - non-overlapping algebraic domain decomposition method (Schur complement method)

Cluster configuration

Features of the nodes :

- 2 Dodeca-core Haswell Intel Xeon E5-2680
- Frequency : 2,5 GHz
- RAM : 128 Go
- Storage : 500 Go
- Infiniband QDR TrueScale : 40Gb/s
- Ethernet : 1Gb/s

3D plane wave in an homogeneous medium



Configuration of the computational domain Ω .

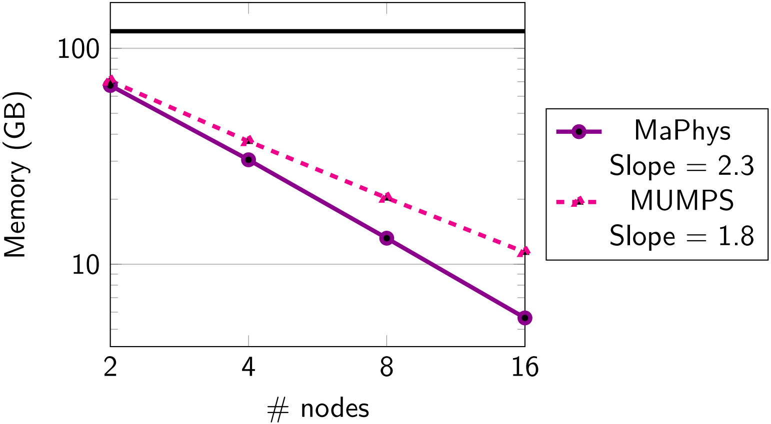
- Physical parameters :
  - $\rho = 1$ kg.m^{-3}
  - $\lambda = 16$ GPa
  - $\mu = 8$ GPa
- Plane wave :

$$u = \nabla e^{i(k_x x + k_y y + k_z z)}$$

- $\omega = 2\pi f, f = 8$ Hz
- Mesh : 21 000 elements

3D Plane wave : Memory consumption

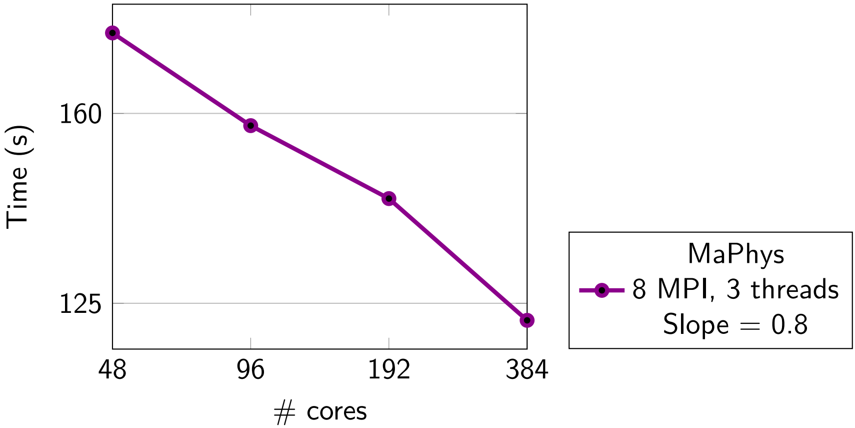
Average memory for one node (8 MPI by node and 3 threads by MPI)



(matrix order = 1 300 000, # nz=300 000 000)

3D Plane wave : Execution time

Execution time for the resolution of the HDG-$\mathbb{P}_3$ system



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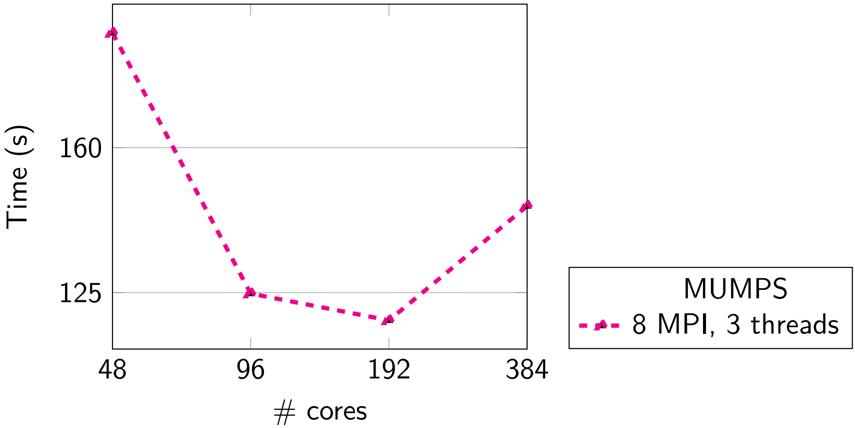

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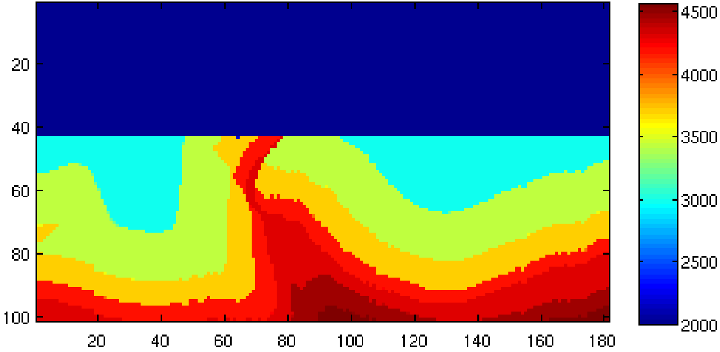
Epati test-case



V_p-velocity model ($m.s^{-1}$), vertical section at $y = 700$ m

Mesh composed of 25 000 tetrahedrons

Epati test-case : Memory consumption

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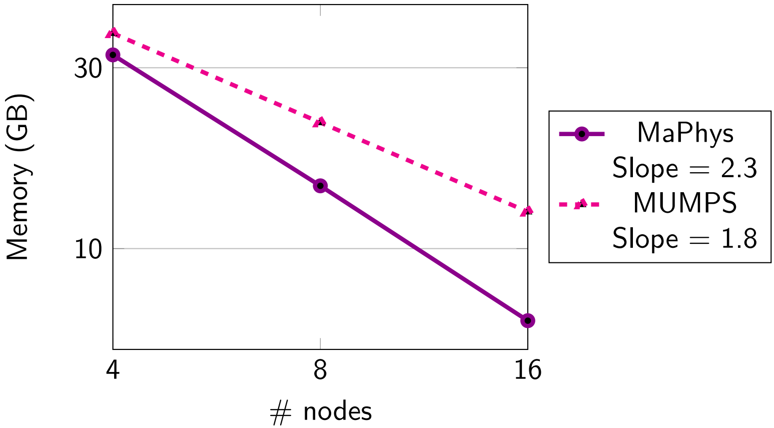

(matrix order = 1 300 000, # nz=365 000 000)

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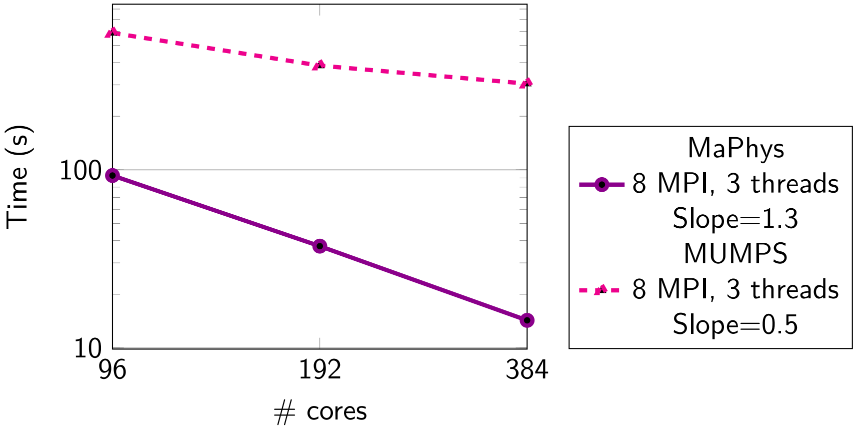

(matrix order = 1 300 000, # nz=365 000 000)

Conclusion-Perspectives

- more detailed analysis of the comparison between solvers
  - larger meshes
  - more powerful clusters
- memory crash test
- extension to elasto-acoustic case

Thank you !